Proofs of Quantumness

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Article

Quantum supremacy using a programmable superconducting processor

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The promise of quantum computers is that certain computational tasks might be executed exponentially faster on a quantum processor than on a classical processor[1]. A fundamental challenge is to build a high-fidelity processor capable of running quantum algorithms in an exponentially large computational space. Here we report the use of a processor with programmable superconducting qubits[2–7] to create quantum states on 53 qubits, corresponding to a computational state-space of dimension 2^{53} (about 10^{16}). Measurements from repeated experiments sample the resulting probability distribution, which we verify using classical simulations. Our Sycamore processor takes about 200 seconds to sample one instance of a quantum circuit a million times—our benchmarks currently indicate that the equivalent task for a state-of-the-art classical

Google, 2019.

Quantum Advantage

Article

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Google, 2019.

RESEARCH

QUANTUM COMPUTING

Quantum computational advantage using photons

Han-Sen Zhong[1,2*], Hui Wang[1,2*], Yu-Hao Deng[1,2*], Ming-Cheng Chen[1,2*], Li-Chao Peng[1,2],

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USTC, 2021.

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[1]*Hefei National Laboratory for Physical Sciences at the Microscale and Department of Modern Physics, University of Science and Technology of China, Hefei 230026, China*

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[4]*QuantumCTek Co., Ltd., Hefei 230026, China*
[5]*Henan Key Laboratory of Quantum Information and Cryptography, Zhengzhou 450000, China*
[6]*Shanghai Institute of Technical Physics, Chinese Academy of Sciences, Shanghai 200083, China*

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Quantum Advantage

Quantum advantage but classically intractable to verify results.

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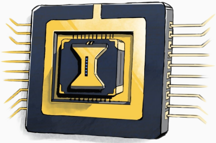
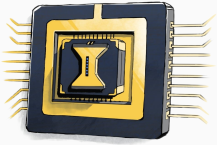
Proofs of quantumness



[0]Thanks to Vivian Uhlir for the figures!

Proofs of quantumness

Verifier



Prover

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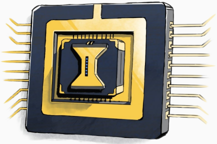

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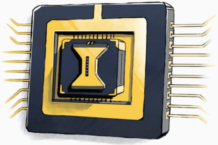

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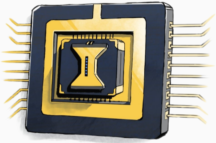

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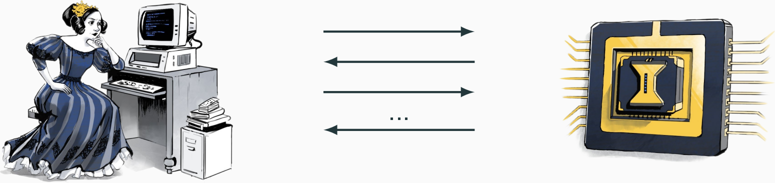

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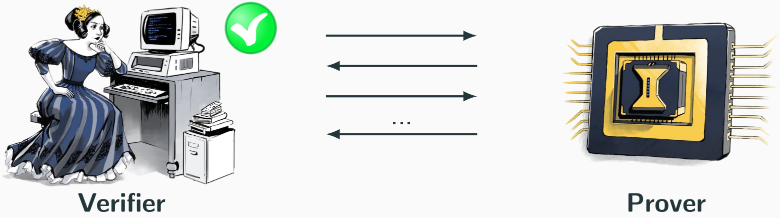

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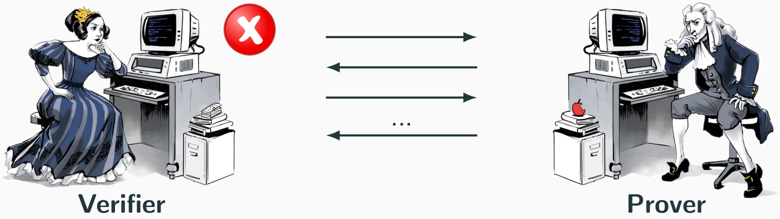

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Proofs of quantumness

Proof of quantumness (PoQ)

Let $\lambda \in \mathbb{N}$ be a security parameter. A PoQ is an interactive protocol between a $\mathrm{poly}(\lambda)$-time *classical verifier* and a $\mathrm{poly}(\lambda)$-time prover, such that

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Soundness is based on a computational assumption.

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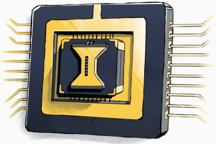
A simple 2-message PoQ



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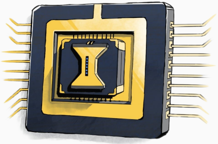
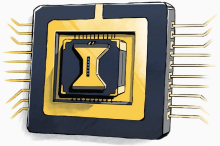


- Pick random λ-bit primes p, q and compute $N = p \cdot q$.

A simple 2-message PoQ

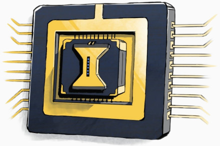
N



- Pick random λ-bit primes p, q and compute $N = p \cdot q$.
- Send N to prover.

A simple 2-message PoQ

N →



- Pick random λ-bit primes p, q and compute $N = p \cdot q$.
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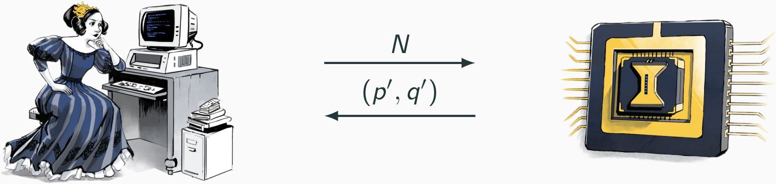
- Factor N using Shor's algorithm.

A simple 2-message PoQ



- Pick random λ-bit primes p, q and compute $N = p \cdot q$.
- Send N to prover.

- Factor N using Shor's algorithm.
- Send factors p', q' to verifier.

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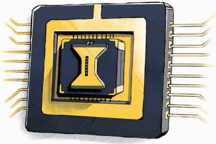
N

(p', q')



- Pick random λ-bit primes p, q and compute $N = p \cdot q$.
- Send N to prover.
- Accept if $N = p' \cdot q'$.

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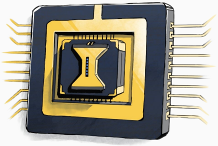
A simple 2-message PoQ

$N \longrightarrow$

$\longleftarrow (p', q')$



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PoQ, assuming *Factoring* $\notin$ BPP.

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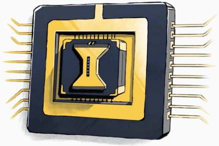

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Can construct such PoQs from any problem, P, such that[1]
$P \in \text{BQP}$, $P \notin \text{BPP}$.

[1]Technically, want $P \notin$ AVBPP.

PoQs with more than 2 messages

Possible to base PoQs on some problem, P, such that $P \notin \text{BQP}$.

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Trapdoor claw-free function (TCF)

We say a family $\{f_\lambda : \mathcal{I} \to \mathcal{O}\}_{\lambda \in \mathbb{N}}$ is a TCF family if:

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Poly-time algorithm that, given $x \in \mathcal{I}$, computes $f_\lambda(x)$.

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Two-to-one

For every $y \in Im(f_\lambda)$, there are *exactly two* $x_0, x_1 \in \mathcal{I}$, $f_\lambda(x_0) = f_\lambda(x_1) = y$.

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Trapdoor

There is a trapdoor t_λ and a poly-time algorithm that, given t_λ and $y \in Im(f_\lambda)$ can compute $x_0, x_1 \in \mathcal{I}$, such that $f_\lambda(x_0) = f_\lambda(x_1) = y$.

[2][Brakerski, Christiano, Mahadev, Vidick, Vazirani '18]

PoQs with more than 2 messages

Strong trapdoor claw-free function (STCF)

We say a family $\{f_\lambda : \mathcal{I} \to \mathcal{O}\}_{\lambda \in \mathbb{N}}$ is a STCF family if it is a TCF and:

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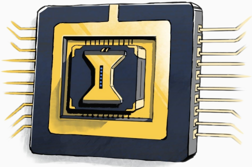
TCFs can be constructed from factoring, discrete-log, Ring-LWE, LWE.

A 4-message PoQ (the BCMVV'18 protocol)



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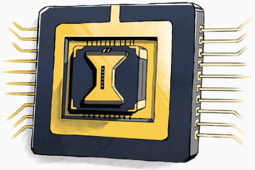
Verifier generates STCF, f_λ, together with trapdoor t_λ.

t_λ

f_λ →



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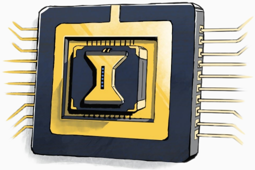

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With probability 1/2.

t_λ

f_λ →

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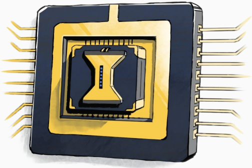
Preimage →



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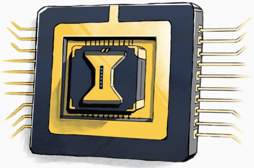
Preimage →

← x



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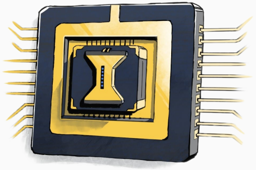
x ←



Verifier accepts if $f_\lambda(x) = y$.

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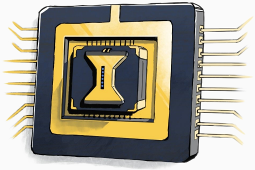
With probability 1/2.

t_λ

f_λ →

$y \in Im(f_\lambda)$ ←

Equation →



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f_λ →

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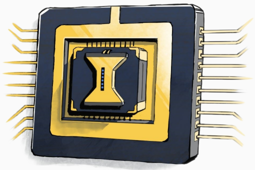
Equation →

← d



Verifier accepts if $d \cdot (x_0 \oplus x_1) = 0$, with $f_\lambda(x_0) = f_\lambda(x_1) = y$.

A 4-message PoQ (the BCMVV'18 protocol)

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With probability 1/2.

Verifier uses t_λ to obtain x_0, x_1 from y and checks the equation.

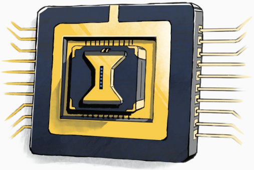
t_λ

f_λ →

← $y \in Im(f_\lambda)$

Equation →

← d



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BCMVV'18 completeness

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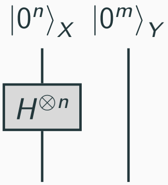

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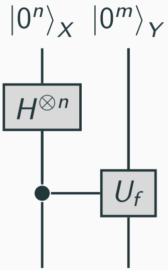

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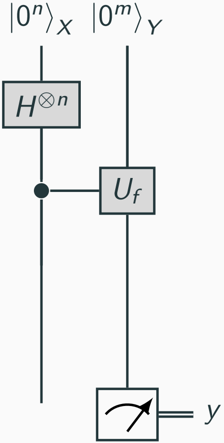

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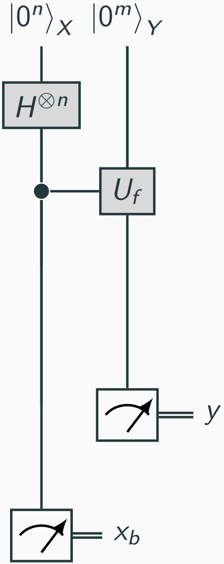

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Preimage case: x_b, $b \leftarrow_U \{0,1\}$

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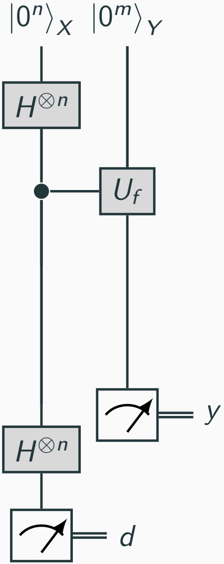

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Equation case: d, $d \cdot (x_0 \oplus x_1) = 0$

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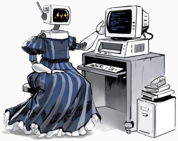

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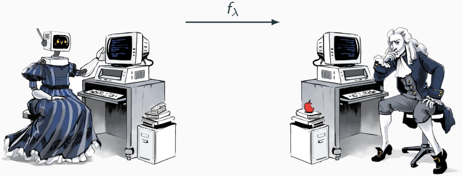

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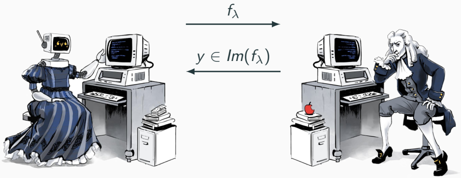

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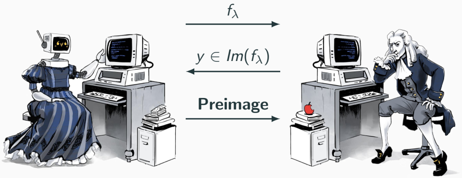

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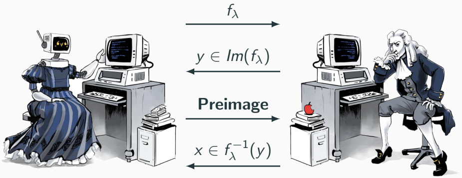

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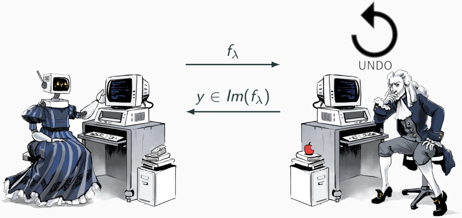

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4. **Rewind**

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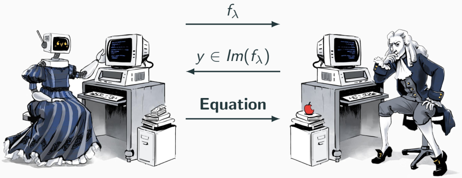

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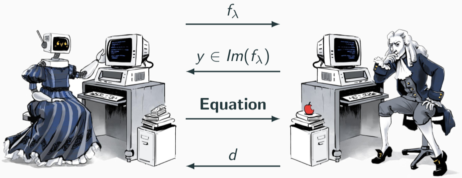

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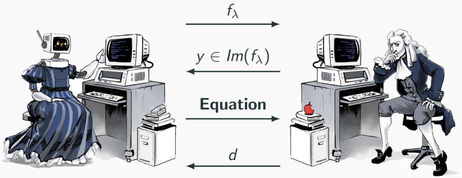

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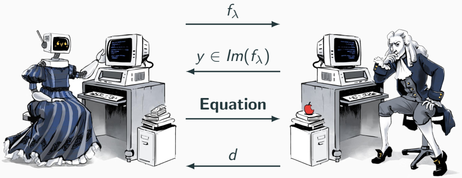

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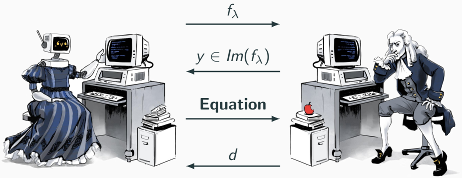

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BCMVV'18 proof of quantumness

BCMVV'18 is a 4-message PoQ with $c(\lambda) = 1$ and $s(\lambda) = 3/4 + \mathrm{negl}(\lambda)$.

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Is the adaptive hardcore bit *necessary*?
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Yes! By "forcing" an equation onto the prover[3].

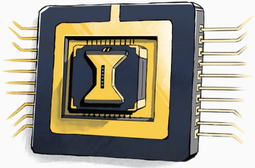
[3] [Kahanamoku-Meyer, Choi, Vazirani, Yao, 2021]

A 6-message PoQ (KCVY'21 protocol)

t_λ

f_λ

$y \in Im(f_\lambda)$



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f_λ

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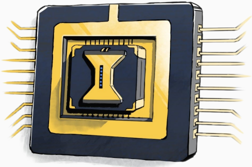


$\frac{1}{\sqrt{2}}(|x_0\rangle + |x_1\rangle)$

A 6-message PoQ (KCVY'21 protocol)

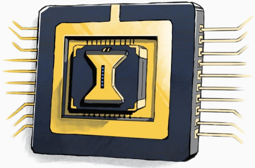
t_λ

f_λ →

$y \in Im(f_\lambda)$ ←

Preimage →

$x \in f_\lambda^{-1}(y)$ ←



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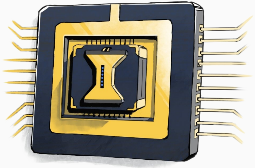
A 6-message PoQ (KCVY'21 protocol)

t_λ

f_λ →

$y \in Im(f_\lambda)$ ←

r →



$r \leftarrow_U \{0,1\}^{\mathrm{poly}(\lambda)}$

$\frac{1}{\sqrt{2}}(|x_0\rangle + |x_1\rangle)$

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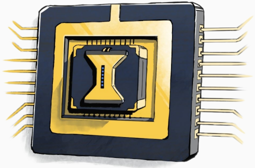
t_λ

f_λ →

$y \in Im(f_\lambda)$ ←

r →



$r \leftarrow_U \{0,1\}^{\mathrm{poly}(\lambda)}$

$\frac{1}{\sqrt{2}}(|x_0\rangle + |x_1\rangle)$

$\frac{1}{\sqrt{2}}(|r \cdot x_0\rangle |x_0\rangle + |r \cdot x_1\rangle |x_1\rangle)$

A 6-message PoQ (KCVY'21 protocol)

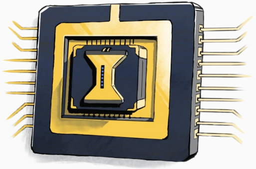
t_λ

$\xrightarrow{f_\lambda}$

$\xleftarrow{y \in Im(f_\lambda)}$

$\xrightarrow{r}$

$\xleftarrow{d}$



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$$\frac{1}{\sqrt{2}}(|x_0\rangle + |x_1\rangle)$$

$$\frac{1}{\sqrt{2}}(|r \cdot x_0\rangle |x_0\rangle + |r \cdot x_1\rangle |x_1\rangle)$$

Hadamard and measure second register

A 6-message PoQ (KCVY'21 protocol)

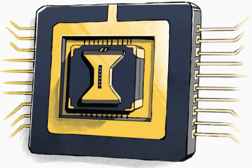
t_λ

f_λ →

$y \in Im(f_\lambda)$ ←

r →

d ←



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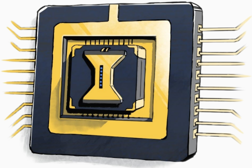

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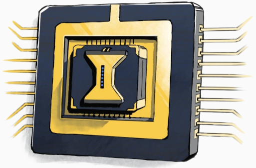

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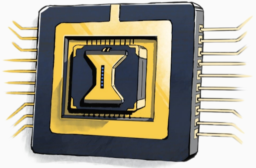

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Measure in basis m, outcome o

A 6-message PoQ (KCVY'21 protocol)

t_λ

$\xrightarrow{f_\lambda}$

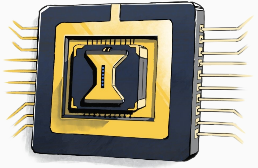
$\xleftarrow{y \in Im(f_\lambda)}$

$\xrightarrow{r}$

$\xleftarrow{d}$

$\xrightarrow{m}$

$\xleftarrow{o}$



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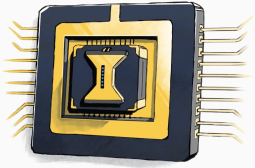

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Measure in basis m, outcome o

Quantum prover succeeds with probability $cos^2(\pi/8) \approx 85\%$

A 6-message PoQ (KCVY'21 protocol)

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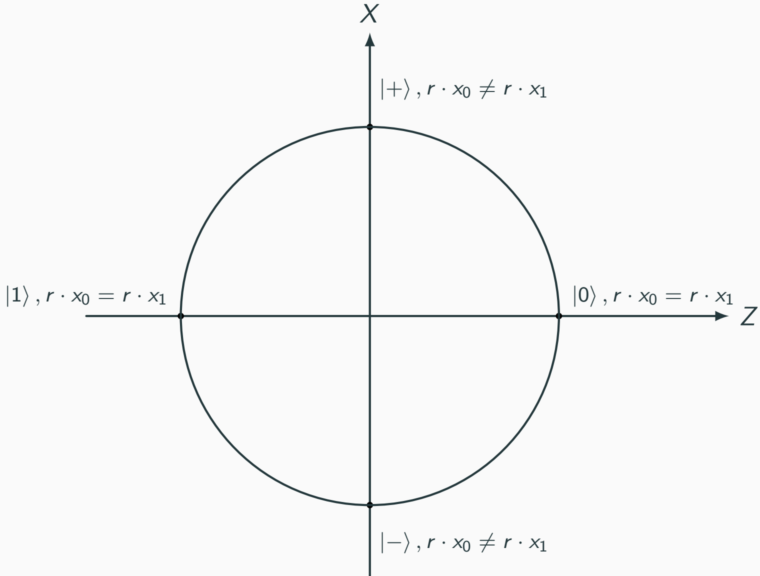

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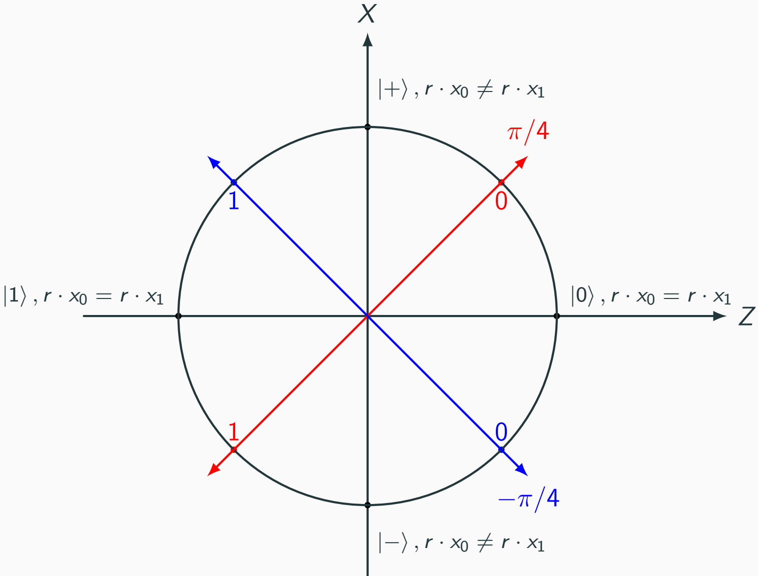

KCVY'21 soundness

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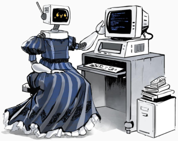

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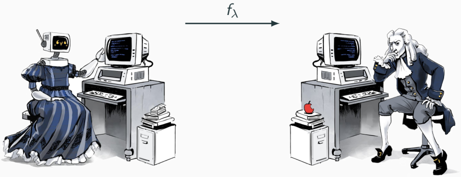

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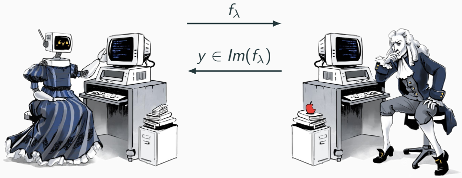

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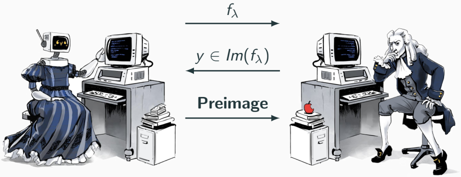

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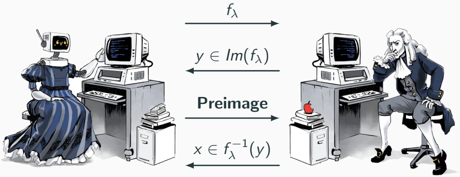

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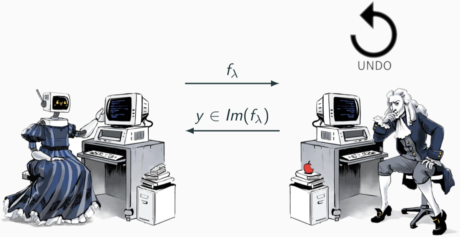

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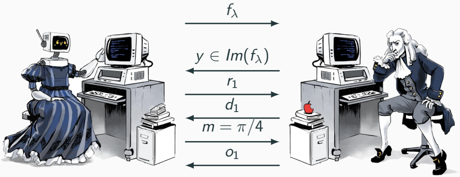

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5. Do Bell test with r_1, $m = \pi/4$.

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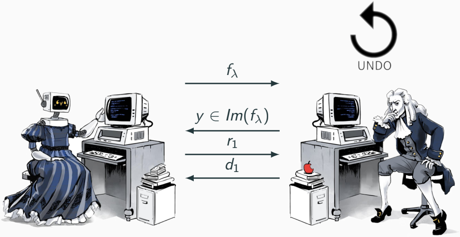

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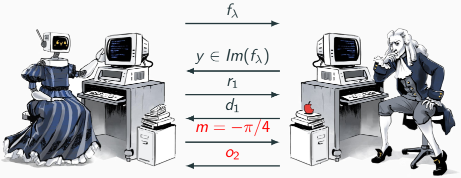

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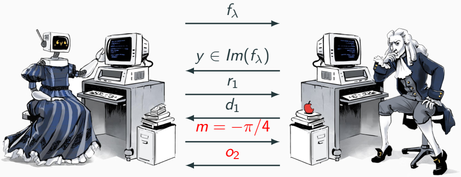

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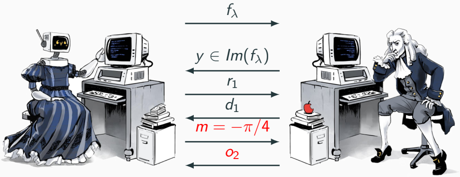

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Outcomes $o_1, o_2, \ldots$ determine bits of $x_0 \oplus x_1$.

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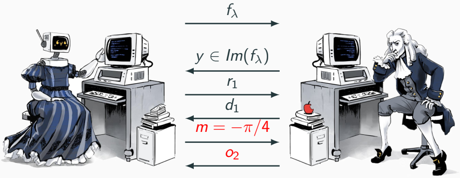

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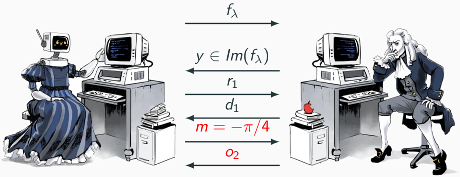

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Can recover both preimages, which contradicts claw-freeness!

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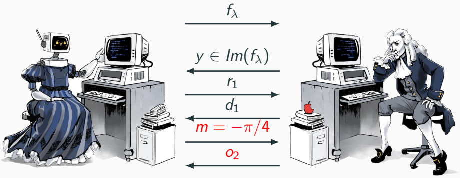

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KCVY'21 proof of quantumness

KCVY'21 is a 6-message PoQ with $c(\lambda) = \frac{1}{2}(1 + cos^2(\pi/8))$ and $s(\lambda) = \frac{1}{2}(1 + 3/4) + \mathrm{negl}(\lambda)$.

KCVY'21 removing the preimage test

Brakerski, Porat and Vidick showed that preimage test can be removed!

KCVY'21 removing the preimage test

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There's another way to do this with a simple modification...

KCVY'21 removing the preimage test

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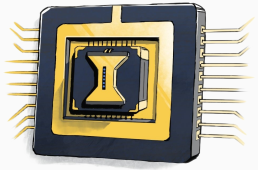
f_λ →

$y \in Im(f_\lambda)$ ←



$\frac{1}{\sqrt{2}}(|x_0\rangle + |x_1\rangle)$

[3] [Gheorghiu, Kahanamoku-Meyer]

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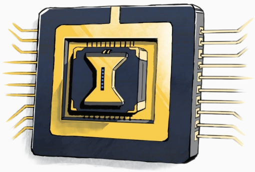
r →



$\frac{1}{\sqrt{2}}(|x_0, 00..0\rangle + |00..0, x_1\rangle)$

[3] [Gheorghiu, Kahanamoku-Meyer]

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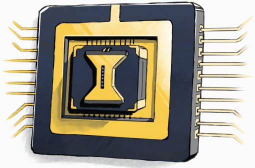

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Accept if prover sends likely o.

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Measure in basis m, outcome o

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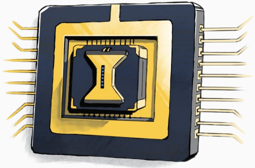

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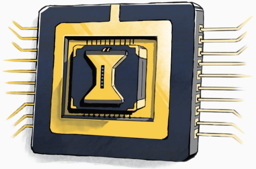

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Measure in basis m, outcome o

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When doing the decoding in the soundness analysis, recover $x_0||x_1$.

[3] [Gheorghiu, Kahanamoku-Meyer]

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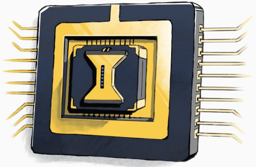

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Preimageless KCVY'21 proof of quantumness

6-message PoQ with $c(\lambda) = cos^2(\pi/8)$ and $s(\lambda) = 3/4 + \mathrm{negl}(\lambda)$.

[3] [Gheorghiu, Kahanamoku-Meyer], [Brakerski, Porat, Vidick]

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- Soundness relies only on claw-free property of TCFs.

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Potential for performing PoQs with non-fault tolerant quantum devices[4]...

[4] Interactive Protocols for Classically-Verifiable Quantum Advantage, Zhu et al. '22.

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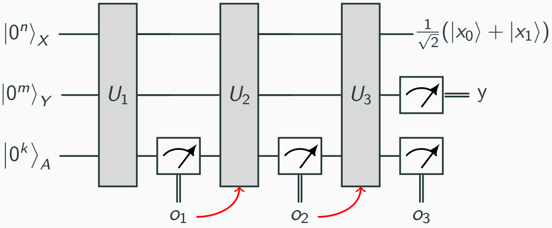

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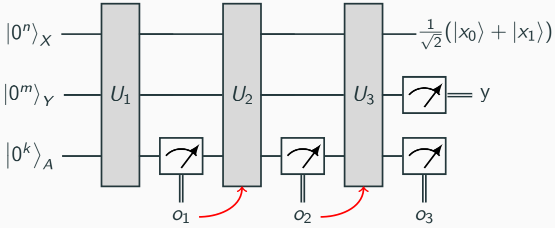

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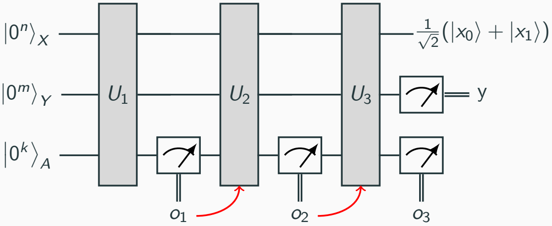

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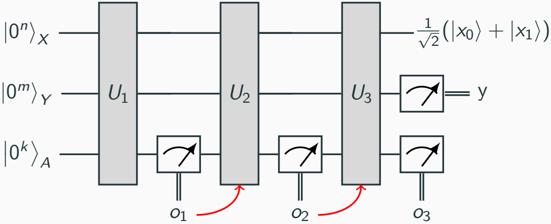

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Circuit width becomes quite high $O(\lambda^8 \log(\lambda))$.

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Non-interactive PoQs - BKVV'20

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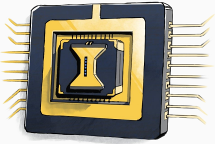

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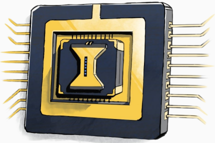

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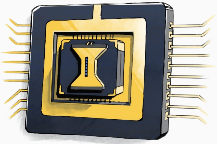

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BKVV'20 construction only relies on claw-freeness for soundness!

Protocol can use TCFs, rather than STCFs.

Non-interactive PoQs - YZ'22

Surprisingly, in ROM, it's possible to remove the use of TCFs![6]

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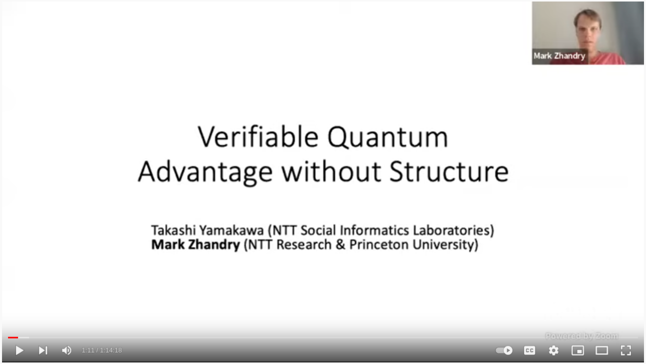

Verifiable Quantum Advantage Without Structure | Quantum Colloquium

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Given a description of C (parity-check matrix), find a codeword $c = (c_1, c_2, ...c_n) \in C$, such that $H(c_1) = H(c_2) = ...H(c_n) = 0$.

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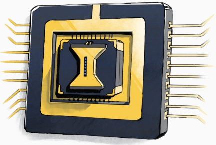

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Are there other PoQs that do not rely on TCFs?

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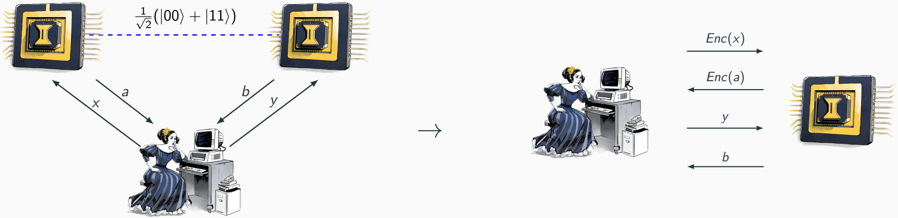

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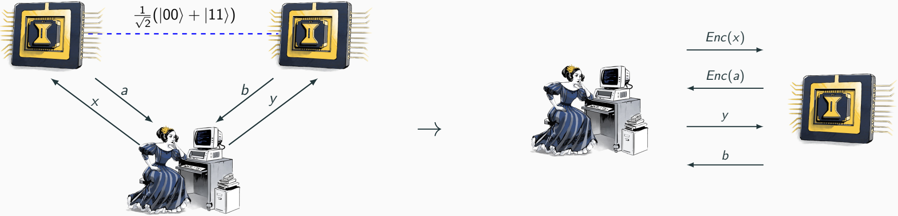

Prover performs quantum strategy of the game *homomorphically*.

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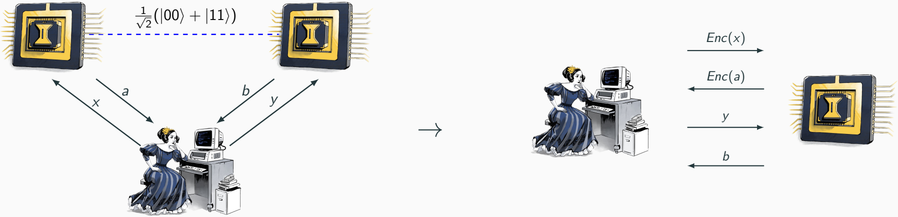

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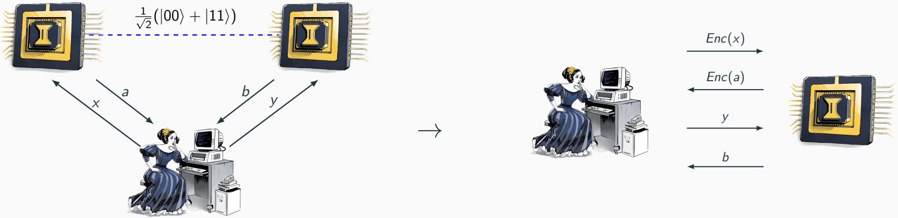

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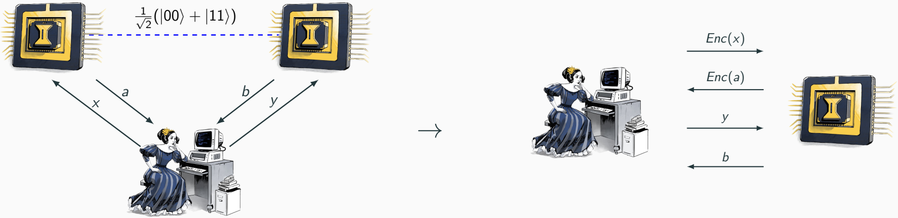

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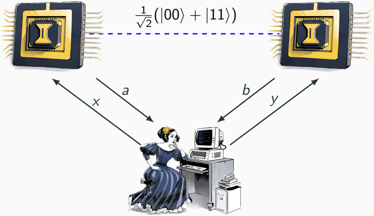

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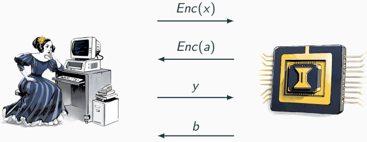

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If non-local game has quantum completeness c and classical soundness s, PoQ will have $c(\lambda) = c$, $s(\lambda) = s + \mathrm{negl}(\lambda)$.

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- Efficient if QFHE with $O(1)$ T gates is efficient!

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Current estimates for quantum advantage demonstration: $\sim 1000 - 2000$ qubits and 10^5 layers of depth.

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